Option Implied Estimation of Risk Aversion – Evidence From India

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Research question: What do 1.5 lakh observations of options data say about risk preferences of Indian investors?

Abstract

Use of risk-neutral density functions to estimate the degree of risk aversion of the representative agent using the options market data has gained tremendous popularity over the last decade. Studying option prices can provide information about the market participants' probability assessment of the future outcome of the underlying asset. This paper explores a nonparametric technique to compute risk-neutral density (RND) functions directly from option index prices and further tests the forecast ability of future densities. This analysis also examines the need to extract the subjective distribution of future asset prices from option prices using utility functions. Results indicate that risk-neutral PDFs are reasonably good forecasts of future densities. The study finds an inverse relation between the computed measure of investor risk aversion and equity risk. Results indicate a phenomenon that is theoretically consistent – there is presence of risk-neutrality among investor.

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1. Introduction

In today's world, it is unequivocal that we must have an understanding of a comprehensive way to measure investor risk preferences. Risk sensitivity focusses its attention towards two aspects – one, the market participants' attitude towards risk; and second, how that risk attitude is measured. This paper focuses on measurement of risk to enable better estimation of investor risk preferences. Prices of any asset class contains complete information about market participant's probability assessment of the outcome of the underlying asset price upon maturity. This information can be extracted using risk-neutral density functions that can provide a forward-looking risk positions of market participants. Theory tells us that investors are risk-averse and rational. Intuitively, this implied that equity market participants are neutral to risk. However, in reality the risk-neutral forecasts may not be true estimators of the investors' risk preferences. The aim of this study computes subjective density functions using utility functions that can yield efficient forecasts of the risk preferences. Can theoretical models of option pricing give a comprehensive and effective measure of risk aversion? The difference between the risk-neutral and utility-adjusted density function yields a measure of relative risk aversion of the representative investor. The aim is to obtain a risk parameter that best fits the realized distribution of option price observations. The contribution of this exercise is to develop the probability density function of the risk-neutral investor and compare it with a utility-adjusted subjective density function with a view to assess their forecast ability. This study adopts the approach of Bliss and Panigritzoglou (2002, 2004) to answer the above questions by using implied volatility to compute a measure of risk aversion.

The primary idea is to extract two density functions – risk-neutral density function and utility-adjusted functions to estimate their forecast abilities of future densities, thus giving a forward-looking estimate of investor preferences in future. This analysis allows us to test the forecast ability of distributions of the ex-post values of the asset prices.

Andersen and Wagener (2002) describe RND as an expression of the market's average position taking that enables investors to compare RND with their own subjective probability assessment of future market outcomes. For instance, the central bank can extract RNDs to trace out the extent of market reaction to any change in interest rates thereby providing a detailed picture of market expectation of future monetary policy decisions[2]. Literature reveals a number of ways to extracting risk-neutral distributions from option prices. Ross (1976) and Cox and Ross (1976) assumed that investors are risk-neutral and utilized the Black-Scholes formula to derive assets expected return. The seminal work by Ross (1976), Cox and Ross (1976) and Breeden and Litzenberger (1978) have been utilized by several papers to extend the analysis to develop more robust approaches to extract RND. One such approach was given by Shimko (1993) in which interpolation of the Black-Scholes implied volatility smile was used to extract RND. Since then, several variants to Shimko's smile interpolation have been developed. Andersen and Wagener (2002), Bliss and Panigritzlou (2004) and Malz (1997) discuss parametric approaches extending Shimko's analysis to extract RND using volatility smile interpolation technique. In addition, Ait Sahalia and Lo (1998) adopt a nonparametric approach to estimate state price density in asset prices thereby developing a method to estimate an arbitrage-free approach to pricing new, complex and illiquid securities. Cheng (2010) develops a framework to compare statistical properties of estimated RNDs among major markets like commodities market, exchange rate market, S&P 500 and the US Treasury

[2] Largely, such research would fall into one of the following three: one, approximating RND by fitting parametric density function to data; two, approximating with a non-parametric technique; or three, by means of theoretical modeling of the return process that generated empirical RND as the maximum probability density for the value of the underlying asset upon maturity.

market. Figlewski (2008) extracts twelve years of daily data and develops a new method of completing the RND with tails drawn from a generalized extreme value distribution, thus taking the analysis a step further. The Black-Scholes option pricing model, developed in 1973, has been used extensively for extraction of implied statistical quantities from underlying assets.

It is important to estimate risk-neutral distribution from option index prices by employing a complete set of information that include past and current options market observations. Options index data are most preferable to study risk preferences as they have fixed maturity dates in future when payoffs are realized[3]. The technique described in this paper works for any asset type subject to availability of good quality data. This paper extracts the risk-neutral probability distribution of future asset prices from option prices by interpolating the Black-Scholes implied volatility smile. The Black-Scholes smile interpolation technique is used to extract risk-neutral probability density for the short term interest rate from option prices on option futures.

This paper is organized as follows: Section 2 details the data and filtering process. Section 3 gives the methodology of extracting PDFs and the theoretical underpinnings of the estimation. Section 4 discusses the forecasting and estimation of subjective or risk-adjusted PDFs. Section 5 provides the estimation results, and finally Section 6 concludes.

2. Data

The dataset includes daily data between the time period January 2001 and June 2015 containing nearly 150,000 observations. Options provide multiple prices for different payoffs for the same underlying asset. The granular data set would enable an analysis of relative risk aversion of investors over varied time horizons. This study aims to employ both power and exponential utility functions to convert the risk-neutral density function into a subjective function. Table 1 provides a descriptive statistics of the options data employed in this analysis. A number of econometric tests are then applied to assess the forecast ability of the risk-neutral as well as subjective density function. Finally, the difference between the two yields us a measure of relative risk aversion (RRA). Several tests are employed to assess which risk aversion parameter that produces subjective density functions can be the best fit for the realized values.

This paper employs options daily contract reports data for the options NIFTY 50 index. The data is obtained from National Stock Exchange (NSE, India). The dataset includes daily data between the time period January 2005 and June 2015. Observations with time-to-maturity of less than one day and implied volatilities greater than 100 percent are dropped. Options market in India expire on the third Thursday of every month for which we consider quarterly expiries i.e. for the months March, June, September and December each year. In order to eliminate excessive noise in the data, we drop all in-the-money strikes and only consider at-the-money and out-of-the-money strikes. We also consider prices only above 1/8 and trades with time to maturity of more than one day.

Options data has emerged most suitable to study risk preferences as they have fixed maturity dates in future when payoffs are realized. In addition, studying the options market provides more insight of investor behaviour over varied time horizons. Options provide multiple prices

[3] In the case of Indian options market, with the exception of Kumar et al (2014), limited research has been done to estimate risk-neutral density (RND).

for different payoffs for the same underlying asset. This feature of options enables computation of a density function for the distribution of possible values. The risk-free rate of interested is obtained from daily closing price of the 3-month MIBOR data, available at NSE website. The dividend yield is taken to be zero.

3. Methodology

The science of extracting RNDs of future asset returns based on option-implied volatility smile is a well-developed practice available for researchers in finance. However, these techniques are empirically and analytically challenging, perhaps also being the reason for lesser application of such a technique. In this paper, we try and describe a simple yet effective approach to estimating RND. It is clear from Malz (2014) who succinctly compiles the methodological issues faced in extracting RNDs and ways to circumvent them, that we can extract RNDs without violating the no-arbitrage conditions. In essence, the computation involves three steps: First, interpolating the volatility smile data so that the data points are not clustered at the end points. Second, utilize the call valuation function (as explained in subsequent sections) to generate the option call value. The third step is to finally numerically differentiate the call valuation function with respect to the strike price to extract the risk-neutral probability density function.

The steps followed in this paper to extract the RND are as under:

1. Use options observation/expiry date pairs and eliminate observations that are too far in or out of money

2. Convert option call prices to Black-Scholes implied volatility (using inverse method)

3. Interpolate the implied volatility data using a fourth degree polynomial spline function

4. Re-convert the interpolated implied volatility curve back to option prices in order to extract the center portion of the RND (using the technique by Breeden and Litzenberger)

The simple technique for computing RNDs follows from Breeden and Litzenberger (1978) who provide the relationship between European option call prices and the RND. The risk-neutral distribution is obtained using the second derivative of call option's value with respect to the exercise price. In absence of arbitrage, the second partial derivative of a continuous European call price function with respect to the strike price of options is closely related with the risk-neutral probability that the future asset price will be no higher than the strike price upon option maturity. This relationship forms the central pivot of the analysis in this paper.

There is a vast and varied methodological literature on extracting RNDs. The simplest approach is to compute histograms. However, these histograms are rough estimations of the underlying density and is based on a discrete approach. Our approach to extracting the risk neutral probability measure follows the technique adopted by Bliss and Panigirtzoglou (2002, 2004). The primary idea behind extracting RND from option prices is that the price of the option contains complete information about market participant's probability assessment of the outcome of the underlying asset price at maturity. The seminal paper by Cox and Ross (1976) revealed that if investors can be assumed to prefer more to less (assumption of local non-satiation) then options can be priced if investors were risk-neutral, independently of their risk preferences. The price of a European style call option can be expressed as the discounted value of the options expected return under the risk neutral distribution:

Equation 1:

$$C = e^{-rT}\hat{E}[max(S - X;0)] = e^{-rT}\int^{\infty} max(S - X;0)f(F)dS$$

where C is the option price, S is the value of the underlying asset, X is the strike price, r is the risk-free interest rate and T is the time to maturity of the option. E here denotes the expectations operator with respect to the risk-neutral density of the future asset price *f(S)*. Equation (1) gives the expression for pricing call options. Unfortunately, it is not possible to empirically determine *f(S)*. This is where the relationship given by Breeden and Litzenberger (1976) becomes significant, that relates European option call prices and the RND. This expression given in equation (2) forms the mathematical foundation for analysis in this paper. The relationship is of the form:

Equation 2:

$$(\partial^2 C(S, X, T, t))/(\partial X^2) = e^{-rT} f(S)$$

The call option price formula is given by:

Equation 3, 4, 5:

$$C = e^{-rT}[S\Phi(d_1) - X\Phi(d_2)]$$

where,

$$(d_1 = ln(S/X) + 1/2\sigma^2 T)/(\sigma\sqrt{(T)})$$

and

$$d_2 = d_1 - \sigma\sqrt{(T)}$$

σ here is the implied volatility, $\varphi(d1)$ and $\varphi(d2)$ are the respective cumulative distribution functions.

The computation of a well-behaved risk-neutral distribution function hinges on a good estimate of implied volatility. In this paper, we follow the approach discussed in Bliss and Panigirtzoglou (2004) who smoothen the implied volatility instead of using the call option prices and then re-convert the smooth implied volatility function into a smoothed price function, which can then be differentiated to achieve the RND, as expressed in equation (2). The Black-Scholes formula is used to extract implied volatilities for European Options. The idea is to generate a series of transformed data that may be easier to interpolate and smoothen[4]. The implied volatility can be expressed in terms of the inverted Black-Scholes expression. The important thing to be noted about adopting the volatility smile method to extract RND is that the Black-Scholes formula used in the process is only used for the purposes of conversion (prices to implied volatility) and reconversion (implied volatility to prices) in order to enable data interpolation. Doing so, in no way, assumes the Black-Scholes formula to be correct. It is therefore safe to say that the approach used in this paper is model-free[5].

4. Forecasting and SDFs

[4] Cubic-spline method is used to smoothen the implied volatility

[5] The implied volatility is computed using the inverted Black-Scholes method (using statistical software MATLAB).

After computing the times series of PDFs for each option observation/expiry date pair, the objective is to test the forecast ability of these PDFs. The PDFs computed in the previous sections is a time series that yields one observation for each pair of option date or expiry. In this section, the forecast ability of the Indian PDFs is tested following the methodology adopted by Bliss and Panigritzoglou (2004). The PDFs so generated is a time series of a single realization X_t for every options observation/expiry. To make a good forecast, the estimated PDF, $\hat{f}_t(.)$ must equal the true PDFs, $f_t(.)$. To test the forecast ability, the null hypothesis thus used is that Xt is independent and is the same as the true PDFs. The null hypothesis is that X_t is independent and to test the condition, $\hat{f}_t(.) = f_t(.)$, the inverse probability function of the PDFs,

$$y_t = \int_{-\infty}^{Xi} ft(u)du$$

such that y_t is uniformly and independently distributed; $y_t \sim$ i.i.d. U(0,1). This paper employs two nonparametric Chi-squared and Kolmogorov-Smirnov tests to check for uniformity of y_t, the inverse probability function.

4.1 Subjective Density Function (SDF)

Several studies conducted for US option market data find that risk-neutral PDFs do not accurately forecast the future densities, thus creating the need to have a risk-adjusted PDF that can prove to be better forecasts. To test the forecast ability of a subjective density function, the first step is to establish a utility function for the representative agent. This technique follows from Ait-Sahalia and Lo (2000) that provide the following relation to estimate a subjective density function.

$$\frac{p(St)}{q(St)} = \zeta(St)$$

Where $p(St)$ is the subjective density function, $q(St)$ is the risk-neutral density function and $\zeta(St)$ represent the pricing kernel or the utility function. Following Bliss and Panigritzoglou (2004), this paper adopts a power-utility function to adjust the PDF. Refer Table 3 for the utility forms used for estimating SDFs.

5. Empirical Results

The methodology used in this paper involved three steps. The first step is to estimate the risk-neutral PDF using spline interpolations technique (Breeden and Litzenberger, 1998 and Panigritzolgou, 2002). In the second step, we assess whether the risk neutral PDFs accurately forecast the distributions of ex-post realizations. Under the null hypothesis of the Chi-square test and K-S tests used, the estimated PDFs is the same as the true PDF. Results reveal that we cannot reject the hypothesis that risk-neutral PDFs give accurate forecasts of the future realizations (p-values are insignificant). (See Table 3)

As risk-neutral PDFs make good forecasts of future ex-post distributions, the forecast ability of the SDF need not be tested. However, in the third step, we estimate a measure of risk aversion (relative risk aversion – RRA) using equation 6. The thirds step in this estimation now turns to the role of SDF.

Once it is established that the risk-neutral PDFs can give good forecasts of the ex-post distributions of option realizations, the need for a risk-adjusted or subjective density function diminishes. However, the relation given in equation 6 can be used to extract a parameter for relative risk aversion. In the third step, this paper computes the utility functions coefficient γ, which also is the parameter of risk aversion[6]. The RRA values computed for Indian markets in this paper is 12.53, compared to 12.7 computed by Ait-Sahalia and Lo originally - the parameters values being near similar.

The coefficient γ helps to obtain the best density forecast of the ex-post distributions. Figure 1 provides the risk aversion calculated as the difference between the risk-neutral and risk-adjusted PDFs, normalized by the mean of the risk-neutral PDF. This risk aversion measure can be compared with the measure of equity risk premium. The global VIX is used as a measure of equity risk and we observe that the risk aversion measure is the inverse of the equity risk (see Figure 2).

6. Conclusion

Prices of any asset class contain complete information about market participant's probability assessment of the outcome of the underlying asset price at maturity. This information can be extracted using risk-neutral density functions. Option prices contain complete information about market expectations of the distribution of the future values of underlying prices of assets. This gives us useful information to extract the risk preferences of investors regarding their attitudes towards risk by studying the distributions of the ex-post values of the asset prices. For this purpose, this paper uses the risk-neutral PDFs and risk-adjusted PDFs to compare which one of the two yields good forecasts of the future densities. This paper reveals that findings in the Indian markets are in tandem with theoretical underpinnings of risk preferences of the representative investor. The paper assumes that investors are rational in their forecast of future distributions of asset prices.

Further the coefficient of risk aversion calculated in this paper is consistent with other measures of risk premia like the VIX. There is an inverse relation between investor risk aversion and equity risk premium.

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[6] The coefficient γ is calculated using Monte Carlo simulations.

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Table 1: Descriptive Statistics of NIFTY50 option index market – India

Time Period (daily): (2005 – 2015) Source: National Stock Exchange, NSE

Variable	Obs	Mean	Std. Dev.	Min	Max
StrikePrice	142972	5008.045	1980.762	1770	11400
Open	142972	204.3876	483.4373	0	5907.95
High	142972	213.3641	491.1783	0	5927.1
Low	142972	194.7736	474.6198	0	5885
Close	142972	456.0268	677.2477	.05	5886.7
LTP	142972	308.9934	598.3649	0	5885
Contracts	142972	18171.64	91631.82	0	3542939
Turnover	142972	49383.36	242362	0	7357745
OpenInterest	142972	482722.3	1238109	0	1.66e+07

Table 2 – Utility functions for risk-adjusted SDFs

$$\text{RRA} = -\frac{StU''(St)}{U'(St)}$$

Utility function	U(St)	U'S(t)	RRA
Power utility	$\frac{S_t^{1-\gamma}}{1-\gamma}$	$S_t^{-\gamma}$	γ
Exponential utility	$\frac{e^{-\gamma St}}{\gamma}$	$e^{-\gamma St}$	γSt

Table 3 – Chi-squared and Kolmogorov-Smirnov *p*-values for risk neutral PDFs

Forecast Horizon	PDF	*p*-value
All 4 weeks	Risk-neutral	0.001***

*** 99% level of significance

Figure 1 – Plot of risk aversion derived from risk-neutral PDFs

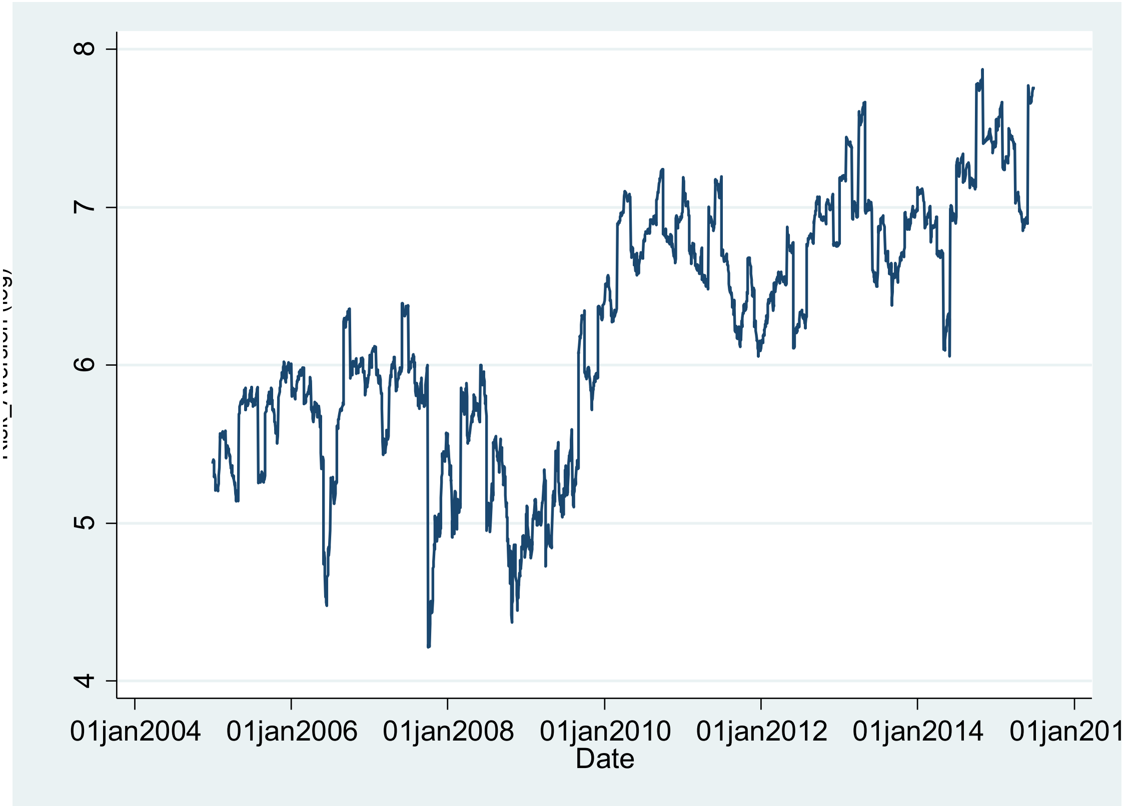

Figure 2 – Inverse relation between risk aversion and equity risk